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On the Black Hole Background of Two-Dimensional String Theory

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Abstract

The classical black hole background of two-dimensional string theory is examined after including the effect of the tachyon field. Keeping all terms upto $O(T^2)$, and making no other approximations, the only consistent classical solution to the resulting dilaton-graviton theory is found to be flat spacetime with a nontrivial dilaton.

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The discovery of a nontrivial dilaton-graviton background of critical two-dimensional string theory with the target space geometry of a 2-d black hole [1] [2] [3] has stimulated much activity in the quantum mechanics of black holes [4]. Subsequently attempts have been made to include the effects of the tachyon in the solution of the beta function equations [5] [6]. However these treatments have thus far been restricted to the asymptotic regime, since the coupled equations are too difficult to handle in general, and can only be compared to the conformal field theory in this regime [7]. In this paper, we will propose a different approach to including the effect of the tachyon with a rather surprising result. We find that upon eliminating the tachyon through its equation of motion, the two dimensional black hole is no longer a consistent classical solution to the resulting dilaton-graviton theory.

We follow the sigma model approach of Mandal, et al [2]. As is well known by now, two-dimensional critical string theory can be viewed as a theory of two-dimensional quantum gravity coupled to $c = 1$ matter [8], or in other words, as a non-critical Polyakov string theory in a single embedding dimension $x^1(\xi)$. In conformal gauge, $g_{ab} = \exp(\eta(\xi))\hat{g}_{ab}$, the conformal field theory on the world-sheet is a theory of two scalars, one being the embedding dimension $x^1(\xi)$ and the other the Liouville field $\eta(\xi)$. The corresponding sigma-model action is

$$S = \frac{1}{4\pi}\int d^2\xi\sqrt{\hat{g}}(\frac{1}{2}\hat{g}^{ab}G_{\mu\nu}(x)\partial_a x^\mu \partial_b x^\nu - \hat{R}^{(2)}\Phi(x) + T(x)) \qquad (1)$$

where $G_{\mu\nu}$, Φ and T denote the low energy excitations of a critical two-dimensional bosonic string: the graviton, the dilaton and the tachyon respectively. Also $x^\mu = (x^1(\xi), \eta(\xi))$. The only propagating mode is the tachyon, which is massless in two dimensions. The graviton and dilaton serve as auxiliary fields that parametrize the background.

The target space equations of motion for these excitations are determined by the conditions of conformal invariance, i.e., the relevant beta-functions are set to zero [9]. As is well-known, these equations can be derived from the target space action

$$I = \int d^2x \exp(-2\Phi)\sqrt{G}(R - 4(\nabla\Phi)^2 + (\nabla T)^2 + V(T) + c) \qquad (2)$$

where $c = -8$ in two dimensions. We will assume that the tachyon potential is quadratic, $V(T) = -2T^2$. The corresponding equations of motion are given by

$$R_{\mu\nu} - 2\nabla_\mu \nabla_\nu \Phi + \nabla_\mu T \nabla_\nu T = 0 \tag{3}$$

$$R + 4(\nabla\Phi)^2 - 4(\nabla^2\Phi) + (\nabla T)^2 + V(T) + c = 0 \tag{4}$$

$$-2\nabla^2 T + 4\nabla\Phi\nabla T + \dot{V}(T) = 0. \tag{5}$$

The black hole solution of [2] is obtained by setting the tachyon field to zero. Then equations (3) and (4) read

$$R_{\mu\nu} - 2\nabla_\mu \nabla_\nu \Phi = 0 \tag{6}$$

$$R + 4(\nabla\Phi)^2 - 4\nabla^2\Phi + c = 0. \tag{7}$$

We will work in the conformal gauge. Then,

$$ds^2 = \exp(\sigma) du dv \tag{8}$$

and the components of the spacetime metric are $G_{uv} = \frac{1}{2}\exp(\sigma)$ and $G_{uu} = G_{vv} = 0$. The only nonvanishing components of the connection, Γ^a_{bc}, are $\Gamma^u_{uu} = \partial_u \sigma, \Gamma^v_{vv} = \partial_v \sigma$. The Ricci tensor is given by $R_{uv} = \partial_u \partial_v \sigma$ and the scalar curvature equals $R = 4\exp(-\sigma)\partial_u\partial_v\sigma$.

Mandal, et al found a non-trivial solution to eqns. (6) and (7),

$$\exp(-2\Phi) = 2uv + a, \tag{9}$$

and

$$\exp(\sigma) = \frac{1}{2uv + a} \tag{10}$$

which implies

$$ds^2 = \frac{du dv}{2uv + a}. \tag{11}$$

For $a \neq 0$ this line element represents a black hole in Kruskal coordinates, with horizon at $uv = 0$ and a curvature singularity at $uv = -\frac{a}{2}$. The string coupling constant $g^2_{st} = \exp(2\Phi)$ grows infinitely large at the location of the curvature singularity.

We will now take into account the dynamics of the tachyon mode in this background. (A linearized analysis of equations (3) to (5) for non-vanishing static tachyons, valid in the asymptotic regime, is carried out in [5], and the extension to non-static tachyons is done in [6].) We begin by manipulating the action. Let us scale the tachyon as follows, $t = \exp(-\Phi)T$. Then the tachyonic action, namely

$$I_T = \int d^2x \exp(-2\Phi)\sqrt{G}((\nabla T)^2 - 2T^2), \tag{12}$$

becomes

$$I_t = \int d^2x\sqrt{G}(G^{\mu\nu}\nabla_\mu t\nabla_\nu t - \frac{1}{4}Rt^2) \tag{13}$$

where we have used the classical equation of motion for the dilaton field (4), and have dropped terms of $O(T^3)$ since we are interested in small fluctuations of the tachyon about its quadratic maximum. Let us eliminate t by using its classical equation of motion

$$(\Box + \frac{1}{4}R)t = 0. \tag{14}$$

The appropriate boundary condition is $t \to 1$ at infinity. Then we can write the solution of the equation of motion:

$$t_{cl} = 1 - \frac{1}{4}\hat{F}R \tag{15}$$

where

$$\hat{F}^{-1} \equiv \Box + \frac{1}{4}R. \tag{16}$$

Note that by using the equation of motion one can express the action I_t as a boundary term

$$I_t = \int d^2x\sqrt{G}\Box t_{cl} \tag{17}$$

or using equation (15)

$$I_t = -\frac{1}{4}\int d^2x\sqrt{G}R + \frac{1}{16}\int d^2x\sqrt{G}R\hat{F}R. \tag{18}$$

This procedure is familiar from the work of Fradkin and Vilkovisky on four-dimensional quantum gravity [10].

In conclusion we obtain the following "effective" action for the coupled graviton-dilaton system:

$$\begin{aligned} I_{eff} &= \int d^2x \exp(-2\Phi)\sqrt{G}(R - 4(\nabla\Phi)^2) - \frac{1}{4}\int d^2x\sqrt{G}R \\ &\quad + \frac{1}{16}\int d^2x\sqrt{G}R\frac{1}{\Box + \frac{1}{4}R}R. \end{aligned} \tag{19}$$

The non-local piece can be evaluated formally as an expansion in powers of $\Box^{-1}R$. In the following, we will approximate $\frac{1}{\Box+\frac{1}{4}R}$ by the leading term in this expansion. The consistency of this approximation will be justified by the result.

By varying I_{eff} we obtain the relevant equations of motion for the graviton dilaton system. First we recall that the Einstein equations are identically satisfied in two dimensions. Also, the variation of $W = \int d^2xR\frac{1}{\Box}R$ is well known to be given by (see for example [11])

$$\begin{aligned}\delta W &= -\sqrt{G}\delta G_{\mu\nu}(2\nabla^\mu\nabla^\nu(\hat{O}R) + \nabla^\mu(\hat{O}R)\nabla^\nu(\hat{O}R) \\ &\quad -G^{\mu\nu}[2R + \frac{1}{2}\nabla_\xi(\hat{O}R)\nabla^\xi(\hat{O}R)]) \end{aligned} \tag{20}$$

where $\hat{O} \equiv \frac{1}{\Box}$. (In the conformal gauge $\Box = 4\exp(-\sigma)\partial_u\partial_v$, so $\hat{O}R = \sigma$.)

Hence we are left with the following set of equations:

$$\partial_u\partial_v\sigma = 2\partial_u\partial_v\Phi + \frac{b}{2}(-3\partial_u\partial_v\sigma + \partial_u\sigma\partial_v\sigma)\exp(2\Phi). \tag{21}$$

$$2(\partial_u^2\Phi - \partial_u\sigma\partial_u\Phi) = -\frac{b}{2}(2\partial_u^2\sigma - (\partial_u\sigma)^2)\exp(2\Phi) \tag{22}$$

$$2(\partial_v^2\Phi - \partial_v\sigma\partial_v\Phi) = -\frac{b}{2}(2\partial_v^2\sigma - (\partial_v\sigma)^2)\exp(2\Phi) \tag{23}$$

$$\partial_u\partial_v\sigma + 4\partial_u\Phi\partial_v\Phi - 4\partial_u\partial_v\Phi = 2\exp(\sigma). \tag{24}$$

Here $b = \frac{1}{8}$. (Of course if we set $b = 0$ we recover the equations of the graviton-dilaton system with $T = 0$.)

The goal is now to check whether the black hole metric satisfies the improved equations of motion derived from the effective action. Note that since we have left the dilaton field unspecified thus far, an *arbitrary* two dimensional black hole metric can, through a coordinate transformation, be brought to the form (10). With no loss of generality then, we can substitute equation (10) in this set of equations. We get

$$\partial_u\partial_v\Phi = -\frac{a}{(2uv+a)^2} - \frac{b}{4}\frac{6a+4uv}{(2uv+a)^2}\exp(2\Phi) \tag{25}$$

$$(\partial_u^2\Phi + \frac{2v}{2uv+a}\partial_u\Phi) = -\frac{b}{4}\frac{4v^2}{(2uv+a)^2}\exp(2\Phi) \tag{26}$$

$$(\partial_v^2\Phi + \frac{2u}{2uv+a}\partial_v\Phi) = -\frac{b}{4}\frac{4u^2}{(2uv+a)^2}\exp(2\Phi) \tag{27}$$

$$\partial_u\Phi\partial_v\Phi - \partial_u\partial_v\Phi = \frac{uv+a}{(2uv+a)^2}. \tag{28}$$

These equations turn out to be consistent only if $a = 0$. This is seen by eliminating $\exp(2\Phi)$ from equations (26) and (27) which implies

$$u^2(\partial_u^2\Phi + \frac{2v}{2uv+a}\partial_u\Phi) = v^2(\partial_v^2\Phi + \frac{2u}{2uv+a}\partial_v\Phi) \tag{29}$$

and then substituting in equation (25) which implies

$$\partial_u\partial_v\Phi = \frac{v}{u}\partial_v^2\Phi + \frac{1}{u}\partial_v\Phi \tag{30}$$

and it therefore follows that $a = 0$.

Now if $a = 0$ the equations (25) to (28) read

$$\partial_u\partial_v\Phi = -\frac{b}{4uv}\exp(2\Phi) \tag{31}$$

$$\partial_u^2\Phi + \frac{1}{u}\partial_u\Phi = -\frac{b}{4u^2}\exp(2\Phi) \tag{32}$$

$$\partial_v^2\Phi + \frac{1}{v}\partial_v\Phi = -\frac{b}{4v^2}\exp(2\Phi) \tag{33}$$

$$\partial_u\Phi\partial_v\Phi - \partial_u\partial_v\Phi = \frac{1}{4uv}. \tag{34}$$

It is easily seen that the solution of the last equation is given by

$$\exp(-\Phi) = A\exp(C_1\ln\sqrt{u} + \frac{1}{C_1}\ln\sqrt{v}) + B\exp(C_2\ln\sqrt{u} + \frac{1}{C_2}\ln\sqrt{v}). \tag{35}$$

By subsituting this expression in (31), (32) and (33) we infer that $AB = b = \frac{1}{8}$ and $C_1 = -C_2 = 1$.

Therefore the improved classical equations of motion yield flat spacetime with the dilaton field given by

$$\Phi = -\ln(A\sqrt{uv} + \frac{b}{A}\frac{1}{\sqrt{uv}}) \tag{36}$$

while the tachyon asymptotically ($t \to 1$) approaches

$$T = \exp(\Phi) = \frac{1}{A\sqrt{uv} + \frac{b}{A}\frac{1}{\sqrt{uv}}} \tag{37}$$

and we recall that $b = \frac{1}{8}$.

By formally letting $b \to 0$, and comparing to the $T = 0$ solution we determine $A = \sqrt{2}$. Our solution again has the property that the string coupling constant $g_{st}^2 = \exp(2\Phi)$ grows infinitely large at $uv = -\frac{b}{A^2} = -\frac{1}{16}$.

What is the implication of our result? It appears that the target space black hole background of two-dimensional string theory could be unstable when the dynamics of the only propagating field in the theory is taken into account, or in other words this background does not represent a true vacuum for the graviton-dilaton system once the nonlinear dilaton-tachyon and graviton-tachyon couplings are considered. (This result was known to Sasha Polyakov [12].) Note that these couplings are not probed in the linearized tachyon equation solved in [5] [6]. Our calculation neglects higher order tachyon self-interactions as well as couplings to the massive modes of the string. It is possible that these are essential to the stability of the black hole solution, as suggested by the conformal field theory analysis [7].

What we find particularly intriguing is the appearance of a non-local term proportional to $\int d^2x\sqrt{G}R\frac{1}{\Box + \frac{1}{4}R}R$ in the spacetime effective action for the graviton-dilaton system. This term is of a similar form to the one proposed years ago by Fradkin and Vilkovisky which when added to the usual Einstein-Hilbert term would represent the appropriate quantum action for four-dimensional quantum gravity (with manifest off-shell conformal invariance) and would yield identical on-shell properties as the Einstein theory (for more details consult [10])! Of course, from the point of view of string theory, this term is a logical possibility in the low-energy effective action that should account for the infrared properties of the gravitational field. In view of recent activity on the problem of black hole evaporation [13] it is perhaps worthwhile reconsidering the role of such non-local terms in the gravitational effective action.

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References

[1] S.Elitzur, A.Forge and E.Rabinovici, Nucl.Phys.**B359** (1991) 581.

[2] G.Mandal, A.M.Sengupta and S.R.Wadia, Mod.Phys.Lett.**A6** (1991) 1685.

[3] E.Witten, Phys.Rev.**D44** (1991) 314.

[4] For a review and further references see J.A.Harvey and A.Strominger, Lectures at TASI'92.

[5] S.DeAlwis and J.Lykken, Phys.Lett.**269B** (1991) 264.

[6] J. G. Russo, University of Texas preprint, UTTG-31-92, December 1992.

[7] S. Chaudhuri and J. Lykken, Fermilab preprint Fermi-92/169-T, to appear in Nucl. Phys. **B** (1992).

[8] A.Dhar, T.Jayaraman, K.S. Narain and S.R.Wadia, Mod.Phys.Lett.**A5** (1990) 863; S.R.Das, S.Naik and S.R.Wadia, Mod.Phys.Lett.**A4** (1989) 1033; S.R.Das, A.Dhar and S.R.Wadia, Mod.Phys.Lett.**A5** (1990) 799; J.Polchinski, Nucl.Phys.**B234** (1989) 123. T.Banks and J.Lykken, Nucl.Phys.**B331** (1990) 173.

[9] C.Callan, D.Friedan, E.Martinec and M.Perry, Nucl.Phys. **B262** (1985) 593.

[10] E.S.Fradkin and G.A.Vilkovisky, Phys.Lett.**73B** 209; Phys.Lett.**77B** 262.

[11] G.A.Vilkovisky, in *Quantum Theory of Gravity* ed. S.Christensen, Bristol, Hilger 1984, pp. 169-209.

[12] A.M.Polyakov, Lectures at Les Houches Summer School, July 1992.

[13] J.G.Russo, L.Susskind and L.Thorlacius, University of Texas and Stanford University preprint, UTTG-19-92 (SU-ITP-92-24), August 1992.